

Soliton Shedding from Airy Pulse in Presence of Higher-Order Dispersive Perturbations

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Received: 26th February, 2024; Accepted: 29th March, 2024; Published: 30th April, 2024

ABSTRACT

In this research paper we have numerically studied the propagation dynamics of Airy pulse in a medium with higher order dispersion and cubic Kerr nonlinearity. Usually, soliton shedding from Airy pulse in presence of anomalous group velocity dispersion (second order) and Kerr nonlinear media is observed. Here, we have reported that higher order dispersion significantly controls the dynamics of emergent soliton. Temporal shift in the emergent soliton is observed in the presence of third as well as fourth order dispersions. Soliton can accelerate or decelerate depending on the sign of these dispersion coefficient.

1. INTRODUCTION

In 1979, a unique solution of time dependent Schrödinger equation was reported in the context of quantum mechanics which is known as Airy wave-packet. This Airy wave-packet remains shape invariant and non spreading during propagation. Importantly, infinite norm of this wave-function makes it impossible to realise experimentally (Berry & Balazs, 1979). In 2007, the concept of finite energy Airy beam (FEAB) has been introduced theoretically and later this beam was experimentally demonstrated in lab (Siviloglou & Christodoulides, 2007; Siviloglou et. al., 2008). Surprisingly, FEAB exhibits the characteristics identical to the ideal Airy function, therefore, further study has been carried out by many different research groups (Efremidis et. al., 2019). The unique features of FEAB have potential applications in optical tweezing, particle trapping and manipulation, curved plasma channel generation and nonlinear optics (Baumgart et. al., 2008; Polynkin et. al., 2009a; Polynkin et. al., 2009b).

The space time duality allows to extend this concept in the time domain as well resulting in the realisation of Airy pulses. The features of Airy pulses are analogous to spatial Airy beam therefore Airy pulse maintains quasi dispersion, self-acceleration, and self-healing characteristics.

Dynamics of Airy pulse has been studied in linear as well as nonlinear media. Airy pulse exhibits tight focusing and inversion in presence of third order dispersion which can be manipulated by the phase modulations (Driben et. al., 2013; Cai et. al., 2018). Airy pulse sheds soliton in the presence of anomalous dispersion and Kerr media and the effect of higher order nonlinearity on the soliton dynamics has been reported recently (Fattal et. al., 2011; Zhang et. al., 2014). The accelerated soliton emerges from the Airy pulse if the input pulse is initially chirped and the sign of chirp parameter decides the acceleration of the soliton

(Purohit et. al., 2021). Recently, unconventional soliton shedding from Airy pulse in presence of negative fourth-order dispersion and Kerr nonlinearity has been reported (Gaur et. al., 2021). In addition, Airy pulses dynamics have been studied in different nonlinear conditions (Gaur et. al., 2022; Gaur & Mishra, 2024a; Gaur & Mishra, 2024b).

In this work, we numerically show the dynamics of the Airy pulse in presence of Kerr nonlinear medium when anomalous dispersion act together with higher-order dispersion i.e. third order dispersion (TOD) and fourth order dispersion (FOD). The dynamics of the conventional soliton from Airy pulse is shown to be manipulated sensitively in the presence of higher order dispersions.

2. THEORETICAL MODEL

Nonlinear Schrödinger equation (NLSE) is employed to model the pulse propagation dynamics in a medium. The NLSE in presence of TOD, FOD and Kerr type nonlinearity can be expressed as

$$i \frac{\partial A}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - \frac{i}{6} \beta_3 \frac{\partial^3 A}{\partial T^3} + \frac{\beta_4}{24} \frac{\partial^4 A}{\partial T^4} + \gamma_1(\omega_0) |A|^2 A = 0, \quad (1)$$

where z and T are the propagation direction and time coordinate in the pulse frame. The parameters β_2 , β_3 and β_4 represent the group velocity dispersion (GVD), TOD and FOD respectively. The pulse envelope is represented by A while γ_1 is known as nonlinear coefficient. The dispersion length $L_{D2} = \frac{T_0^2}{|\beta_2|}$ and nonlinear length $L_{NL} = \frac{1}{\gamma_1 P_0}$ are defined for making parameters dimensionless. The other dimensionless parameters are defined as $\xi = \frac{z}{L_{D2}}$, $\tau = \frac{T}{T_0}$ and $U = \frac{A}{\sqrt{P_0}}$, where T_0 and P_0 is known as pulse width and pulse peak power respectively. The dimensionless NLSE is solved numerically employing the split step Fourier Transform method. The NLSE in dimensionless form is expressed as

$$\frac{\partial U}{\partial \xi} + \frac{i}{2} \text{sgn}(\beta_2) \frac{\partial^2 U}{\partial \tau^2} - \frac{1}{6} \delta_3 \frac{\partial^3 U}{\partial \tau^3} - \frac{i}{24} \delta_4 \frac{\partial^4 U}{\partial \tau^4} - i N^2 |U|^2 U = 0, \quad (2)$$

where $\delta_3 = \frac{\beta_3}{T_0 |\beta_2|}$, $\delta_4 = \frac{\beta_4}{T_0^2 |\beta_2|}$, and $N^2 = \frac{L_{D2}}{L_{NL}}$.

The normalised input pulse envelope is represented as

$$U(\tau, 0) = f(a) \text{Airy}(\tau) \exp(a\tau) \quad (3)$$

where a is known as truncation parameter, introduced to make Airy function physically realisable. Parameter $f(a)$ is the truncation dependent parameter that keeps the input Airy pulse intensity at 1. In our simulation the numerical value of a is chosen to be 0.1.

3. RESULTS

In order to investigate the effect of TOD and FOD on soliton shedding, we solve eqn. (2) numerically where the input pulse as described in eqn. (3) is considered as initial condition. Temporal and spectral evolution of Airy pulse are shown respectively in Figure 1 (a) and (b) when only anomalous dispersion and Kerr nonlinearity is present in the medium. Parameter $N = 1$ denotes that dispersion length and nonlinear length is same for the input pulse therefore the dispersive effects are exactly compensated by the nonlinear effects thus the

soliton emerges from Airy pulse. Initially, sub lobes of the Airy pulse collide, and pulse is compressed before soliton shedding and the accelerating wavefront succumbs to dispersion. Here, the downchirp induced by the anomalous dispersion is compensated by the upchirp induced by the Kerr type nonlinearity therefore soliton shedding is observed as shown in Figure 1 (a). A small change in the pulse spectrum is noticed due to the nonlinearity as shown in fig. 1 (b). The value of parameter N can be increased by increasing the input pulse power. For $N = 1.5$, temporal evolution is illustrated in Figure 2 (a) which shows that emergent soliton becomes intense for high intensity input pulse. The variation of intensity maximum of the emergent soliton for different value of N is depicted in Figure 2 (b). As the value of N increases, soliton intensity and number of oscillations in the intensity is increased but the periodicity is decreased.

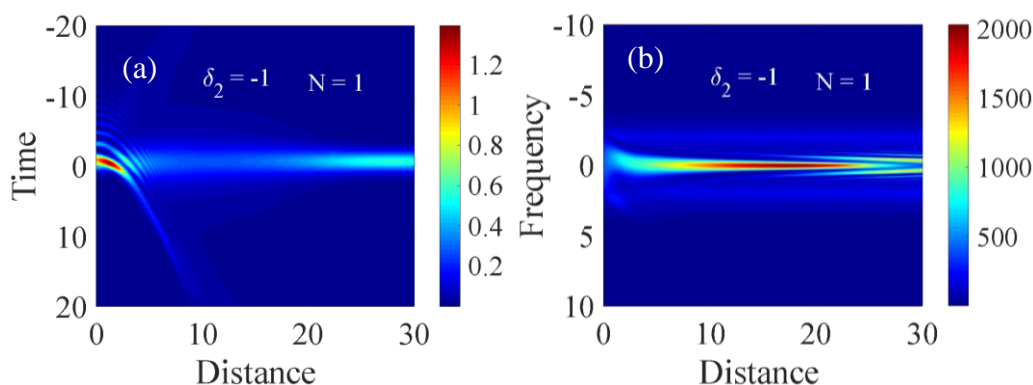


Figure 1 (a): Temporal and (b) spectral evolution of Airy pulse in anomalous dispersion for $N = 1$

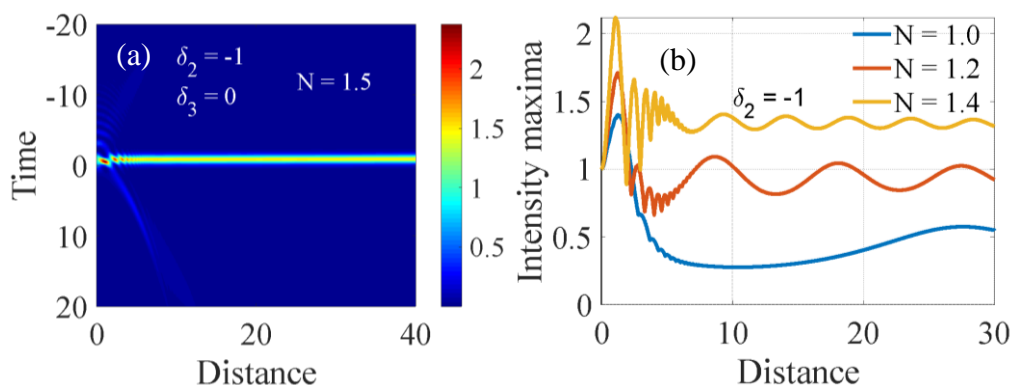
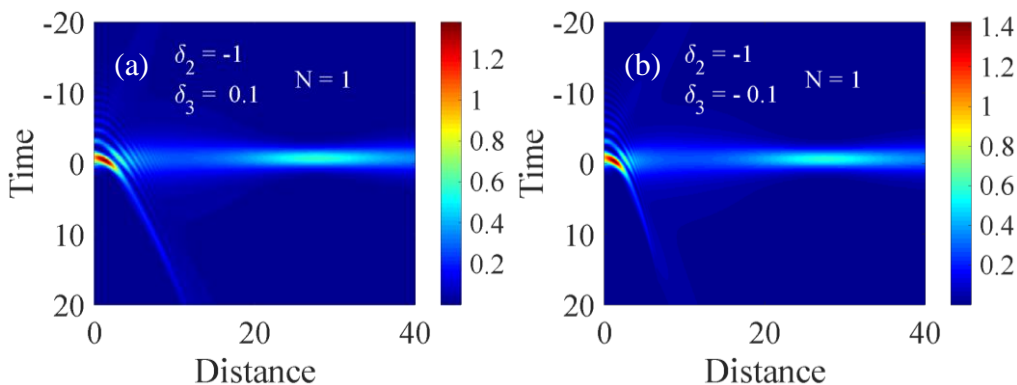


Figure 2 (a): Temporal evolution of Airy pulse in anomalous dispersion for $N = 1.5$ and (b) intensity maxima of the emergent soliton with propagation distance



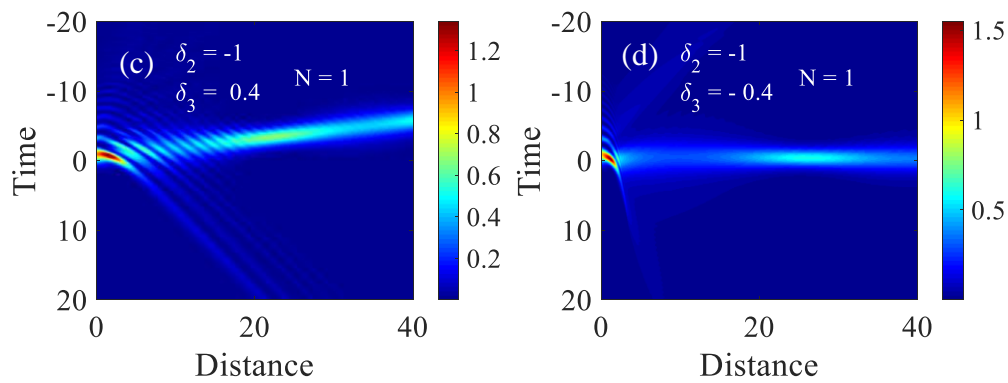


Figure 3: Temporal evolution of Airy pulse in anomalous dispersion ($\delta_2 = -1$) and (a)-(c) positive TOD (b)-(d) negative TOD for $N = 1$

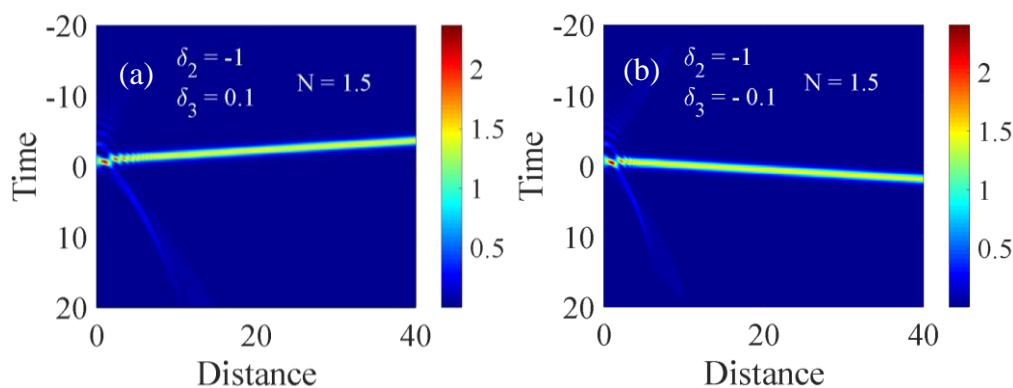


Figure 4: Temporal evolution of Airy pulse in anomalous dispersion ($\delta_2 = -1$) and (a) positive TOD (b) negative TOD for $N = 1.5$

The effects of third order dispersive perturbation on soliton shedding is examined by considering nonzero value of δ_3 in our simulation. The Figure 3 (a) and (b) show that soliton shedding remains unaffected by the small perturbation as introduced by the third order dispersion whereas the large value of δ_3 significantly alter the dynamics of emergent soliton as shown in Figure 3 (c) and 3 (d). For $\delta_2 = -1$ and $\delta_3 = 0.4$, emergent soliton shifts towards the leading edge and the tail of the Airy pulse witness relatively larger acceleration for $\delta_3 = -0.4$. Note that for $\delta_3 = -0.4$, the Airy pulse flips and soliton does not exhibit acceleration (Driben et. al., 2013). Further, we examine the effect of TOD in presence of higher nonlinearity $N = 1.5$. We would like to note that unlike to the previous case here a large temporal shift in the emergent soliton appears even for small value of δ_3 for both of its signs for larger ($N = 1.5$). Emergent soliton shifts towards the leading edge for $\delta_3 = 0.1$ as shown in Figure 4 (a) while the soliton shifts towards the trailing edge for $\delta_3 = -0.1$ as illustrated in fig. 4 (b).

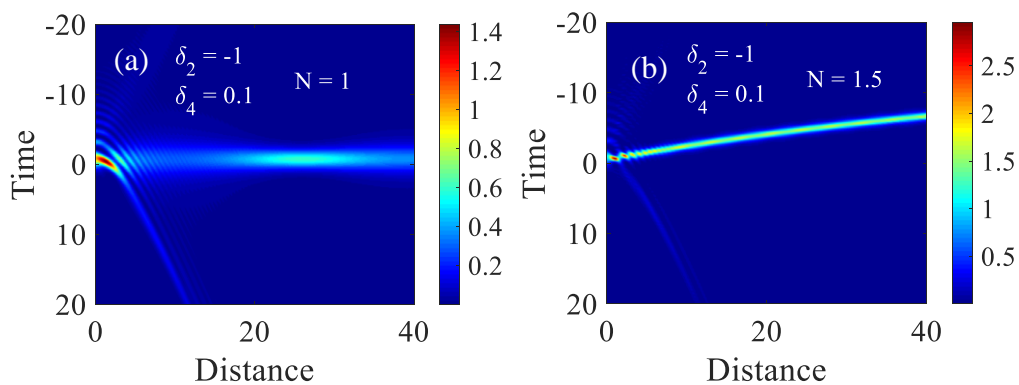


Figure 5: Temporal evolution of Airy pulse in anomalous dispersion ($\delta_2 = -1$) and positive FOD ($\delta_4 = 0.1$) for (a) $N = 1$ and (b) $N = 1.5$

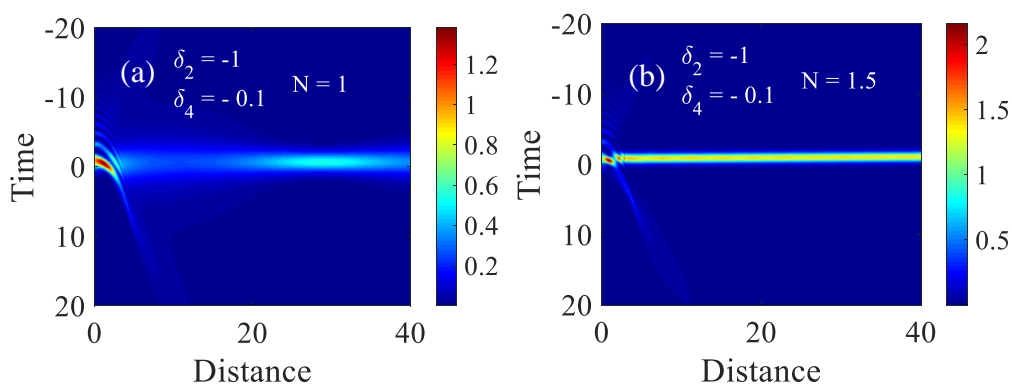


Figure 6: Temporal evolution of Airy pulse in anomalous dispersion ($\delta_2 = -1$) and negative FOD ($\delta_4 = -0.1$) for (a) $N = 1$ and (b) $N = 1.5$

Furthermore, we have also observed the effect of FOD on soliton shedding. For this, we deliberately off the effect of TOD and consider only GVD and FOD in the medium. For $N = 1$, small value of FOD does not significantly alter the soliton dynamics as shown in Figure 5 (a) and 6 (a). For larger value of nonlinear parameter $N = 1.5$, emergent soliton dynamics starts to depend on the sign of the FOD term. In case of positive FOD, soliton shifts towards the leading edge while this acceleration is not observed in case of negative FOD as shown in Figure 5 (b) and 6 (b) respectively. For negative FOD, the soliton does shift towards the leading edge but for even larger nonlinearity (Figure not given).

4. CONCLUSION

In summary, the emergent soliton dynamics has been found to be strongly influenced by the presence of higher order dispersion coefficients. In a weaker nonlinear medium with smaller values of TOD and FOD coefficients, soliton dynamics remains almost unaffected. For a medium with stronger nonlinearity, even smaller values of higher order dispersion coefficients change the soliton dynamics. Depending on the sign of TOD, the shedded soliton shifts either to leading edge or trailing edge of the Airy pulse. Further, we have studied the effects of FOD term on the soliton shift and it has been observed that FOD shifts soliton towards the leading edge.

STATEMENTS AND DECLARATIONS

All authors of this article have made substantial contributions towards the conception and design of the work, acquisition, analysis, and interpretation of data; as well as for the preparation of draft and editing of the manuscript and approved the final version for submission.

FUNDING

This work received no specific funding to any of the Authors.

CONFLICT OF INTEREST

The authors further declare that there are no financial or non-financial interests that are directly or indirectly related to the work submitted for publication; the work was not supported by any governmental or non-governmental financial institutions; and the authors declare no conflicting interests.

REFERENCES

- Baumgart, J., Mazilu, M., & Dholakia, K. (2008). Optically mediated particle clearing using Airy wavepackets. *Nature Photonics*, 2, 675–678. <https://doi.org/10.1038/nphoton.2008.201>
- Berry, M. V., & Balazs, N. L. (1979). Nonspreading wave packets. *American Journal of Physics*, 47, 264–267. <https://doi.org/10.1119/1.11855>
- Cai, W., Wang, L., & Wen, S. (2018). Role of third-order dispersion in chirped Airy pulse propagation in single-mode fibers. *Optics Communications*, 413, 24–29. <https://doi.org/10.1016/j.optcom.2017.12.031>
- Driben, R., Hu, Y., Chen, Z., Malomed, B. A., & Morandotti, R. (2013). Inversion and tight focusing of Airy pulses under the action of third-order dispersion. *Optics Letters*, 38, 2499–2501. <https://doi.org/10.1364/OL.38.002499>
- Efremidis, N. K., Chen, Z., Segev, M., & Christodoulides, D. N. (2019). Airy beams and accelerating waves: an overview of recent advances. *Optica*, 6, 686–701. <https://doi.org/10.1364/OPTICA.6.000686>
- Fattal, Y., Rudnick, A., & Marom, D. M. (2011). Soliton shedding from Airy pulses in Kerr media. *Optics Express*, 19, 17298–17307. <https://doi.org/10.1364/OE.19.017298>
- Gaur, D. S., Purohit, A., & Mishra, A. K. (2021). Soliton shedding from Airy pulses in a highly dispersive and nonlinear media. *Journal of the Optical Society of America B*, 38, 3729–3736. <https://doi.org/10.48550/arXiv.2107.13733>
- Gaur, D. S., & Mishra, A. K. (2022). Impact of external potential in presence of harmonic potential. *Journal of Optics*, 24, 065504. <https://doi.org/10.1088/2040-8986/ac6c92>
- Gaur, D. S., Mishra, & A. K. (2024a). Ballistic dynamic of emergent soliton from Airy pulse in a medium with linear optical potential. *Optics and Laser Technology*, 168, 109996. <https://doi.org/10.1016/j.optlastec.2023.109996>
- Gaur, D. S., Purohit, A., & Mishra, A. K. (2024b). Trapping and guiding soliton from Airy pulse. *Optics Communications*, 556, 130300. <https://doi.org/10.1016/j.optcom.2024.130300>
- Polynkin, P., Kolesik, M., & Moloney, J. V. (2009a). Filamentation of femtosecond laser Airy beams in water. *Physics Review Letters*, 103, 123902. <https://doi.org/10.1103/PhysRevLett.103.123902>

- Polynkin, P., Kolesik, M., Moloney, J. V., Siviloglou, G. A., & Christodoulides, D. N. (2009b). Curved plasma channel generation using ultraintense Airy beams, *Science*, 324, 229–232. <https://doi.org/10.1126/science.1169544>
- Purohit, A., Gaur, D. S., Mishra, & A. K. (2021). Dynamics of a chirped Airy pulse in a dispersive medium with higher order nonlinearity. *Journal of the Optical Society of America B*, 38, 3608-3615. <https://doi.org/10.1364/JOSAB.439227>
- Siviloglou, G. A., & Christodoulides, D. N. (2007). Accelerating finite energy Airy beams. *Optics Letters*, 32, 979–981. <https://doi.org/10.1364/ol.32.000979>
- Siviloglou, G. A., Broky, J., Dogariu, A., & Christodoulides, D. N. (2008). Ballistic dynamics of Airy beams. *Optics Letters*, 33, 207–209. <https://doi.org/10.1364/OL.33.000207>
- Zhang, L., Zhang, J., Chen Y., Liu A., & Liu G. (2014). Dynamic propagation of finite-energy Airy pulses in the presence of higher-order effects. *Journal of the Optical Society of America B*, 31, 889–897. <https://doi.org/10.1364/JOSAB.31.000889>

How to cite this article: Gaur, D. S., & Mishra, A. K. (2024). Soliton Shedding from Airy Pulse in Presence of Higher-Order Dispersive Perturbations. *Vantage: Journal of Thematic Analysis*, 5(1), 60-66

DOI: <https://doi.org/10.52253/vjta.2024.v05i01.07>

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